	Cartesian curve	Parametric curve	Polar curve
Surface area of revolution (S)	$2\pi \int_a^b y \frac{ds}{dx} dx$ (about the x-axis) $2\pi \int_c^d x \frac{ds}{dy} dy$ (about the y-axis)	$2\pi \int_{t_1}^{t_2} y \frac{ds}{dt} dt$ (about the x-axis) $2\pi \int_{t_1}^{t_2} x \frac{ds}{dt} dt$ (about the y-axis)	$2\pi \int_{\theta_1}^{\theta_2} r \sin\theta \frac{ds}{d\theta} d\theta$
Volume of revolution (V)	$\pi \int_a^b y^2 dx$ (about the x-axis) $\pi \int_c^d x^2 dy$ (about the y-axis)	$\pi \int_{t_1}^{t_2} y^2 \frac{dx}{dt} dt$ (about the x-axis) $\pi \int_{t_1}^{t_2} x^2 \frac{dy}{dt} dt$ (about the y-axis)	$\frac{2\pi}{3} \int r^3 \sin\theta \, d\theta$ (about the line $\theta = 0$ or x-axis) $\frac{2\pi}{3} \int r^3 \cos\theta \, d\theta$ (about the line $\theta = \pi/2$ or y-axis)

UNIT - VI DIFFERENTIAL EQUATIONS (D.E)

Methods of solving the D.E at a glance

General form of the D.E : $M(x, y)\,dx + N(x, y)\,dy = 0$

	Form of the D.E	Method of solving / solution
I	Variables separable form (Recapitulation)	
1.	$f(x)g(y)dx + F(x)G(y)dy = 0$	Divide by $g(y)F(x)$ and integrate.
2.	$\frac{dy}{dx} = f(ax+by+c)$	Put $ax+by+c = t$
3.	$\frac{dy}{dx} = \frac{(ax+by)+c}{k(ax+by)+c'}$	Put $ax+by = t$
II	**Homogeneous form**	
1.	$M(x, y)$ and $N(x, y)$ are homogeneous functions of the same degree with or without the involvement of terms with (y/x)	Write the D.E in the form $\frac{dy}{dx} = -\frac{M(x, y)}{N(x, y)}$ and put $y = vx$

	If homogeneous functions are involved with x/y	Write $\frac{dx}{dy} = -\frac{N(x, y)}{M(x, y)}$ and put $x = vy$
2.	$\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'}, \frac{a}{a'} \neq \frac{b}{b'}$	Put $x = X+h, y = Y+k$ With proper choice of h and k the D.E reduces to a homogeneous D.E in X and Y. Put $Y = VX$ and solve.
III	**Exact form**	
1.	$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ must be satisfied.	$\int M\,dx + \int N(y)\,dy = c$ is the solution.
2.	When $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ then	Multiply the D.E with I.F to make it exact.
	(a) If $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$	$e^{\int f(x)\,dx}$ is the I.F
	$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = g(y)$	$e^{-\int g(y)\,dy}$ is the I.F
	(b) $y f(xy)\,dx + x g(xy)\,dy = 0$	$\frac{1}{Mx - Ny}$ is the I.F
	(c) M and N involving terms of the form $x^a y^b$	$x^a y^b$ is the I.F where a and b are found such that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
3.	Identifying the standard exact differentials and putting the D.E in the form $c_1 d\,[f_1(x, y)] + c_2 d\,[f_2(x, y)] + \cdots = 0$	$c_1 f_1(x, y) + c_2 f_2(x, y) + \cdots = c$ is the solution on integration
IV	**Linear form**	
1.	$\frac{dy}{dx} + Py = Q$ where P and Q are functions of x.	**Solution :** $y e^{\int P\,dx} = \int Q e^{\int P\,dx} dx + c$
2.	$\frac{dx}{dy} + Px = Q$ where P and Q are functions of y.	**Solution :** $x e^{\int P\,dy} = \int Q e^{\int P\,dy} dy + c$
3.	$f'(y)\frac{dy}{dx} + f(y)P = Q$ where $P = P(x)$ and $Q = Q(x)$	Put $f(y) = t$ and differentiate $w.r.t$ x.
4.	$f'(x)\frac{dx}{dy} + f(x)P = Q$ where $P = P(y)$ and $Q = Q(y)$	Put $f(x) = t$ and differentiate $w.r.t$ y.

5.	$\frac{dy}{dx} + Py = Q y^n$ where $P = P(x)$ and $Q = Q(x)$	Divide by y^n and put $y^{1-n} = t$ and diff. w.r.t x.
6.	$\frac{dx}{dy} + Px = Q x^n$ where $P = P(y)$ and $Q = Q(y)$	Divide by x^n and put $x^{1-n} = t$ and differentiate $w.r.t$ y.

Inverse of a square matrix A

$$A^{-1} = \frac{1}{|A|}(Adj\,A)$$

Normal form / canonical form of a matrix

(i) I_r (ii) $[I_r, 0]$ (iii) $\begin{bmatrix} I_r \\ 0 \end{bmatrix}$ (iv) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$

where I_r is the identity matrix of order r.

Rank of a matrix $A : \rho(A)$

- $\rho(A)$ = The number of nonzero rows in the row echelon/normal form of A

Given a matrix A there always exist non singular matrices P and Q such that PAQ is in the normal form.

Consistency of a system of equations $AX = B$

- The system is consistent if $\rho[A] = \rho[A:B]$
- The system will have *unique solution* if $\rho[A] = \rho[A:B] = n$, n being the number of unknowns.
- The system will have *infinite / many solutions* if $\rho[A] = \rho[A:B] = r < n$
- The system is *inconsistent* (does not have solution) if $\rho[A] \neq \rho[A:B]$
- The system $AX = 0$ will have trivial solution $(x_1 = 0 = x_2 = \cdots x_n)$ if $\rho[A] = \rho[A:B] = n$. The system will have nontrivial, infinite number of solutions if $\rho[A] = \rho[A:B] = r < n$
- *Gauss elimination method* : In $[A:B]$, A is reduced to the upper triangular form.
- *Gauss Jordan method* : In $[A:B]$, A is reduced to the diagonal form.

Eigen values and Eigen vectors of a square matrix A

- $|A-\lambda I| = 0$ will give the charecteristic equations of A. Eigen vlues are obtained on solving this equation.
- $[A-\lambda I][X] = [0]$ represents a system of equations. On solving this system of equations, eigen vector corresponding to each of the eigen value λ is obtained.

Similarity of matrices and Diagonalisation

- If $B = P^{-1}AP$ then B is said to be similar to A, where A and B are square matrices and P is a nonsingular matrix.
- $P^{-1}AP = D = Diag(\lambda_1, \lambda_2, \lambda_3)$ where $\lambda_1, \lambda_2, \lambda_3$ are the eigen values of A (A *being a square matrix of order* 3)
- $A^n = PD^nP^{-1}$ where $D^n = Diag(\lambda_1^n, \lambda_2^n, \lambda_3^n)$

Quadratic Form (Q.F)

- Quadratic form can be reduced to canonical form (*sum of squares*) by congruent / orthogonal transformation.

  * Rank (r) of the Q.F = Rank of the matrix of the Q.F
  * Index (p) of the Q.F = Number of positive terms in the canonical form
  * Signature of the Q.F = The difference between the number of positive and negative terms in the canonical form

Nature of the Quadratic Form

	Condition	Nature of Q.F	Canonical form	Remark on canonical form
1.	$r = n, p = n$	Positive definite	$y_1^2 + y_2^2 + \cdots + y_n^2$	Only positive terms (n terms).
2.	$r = n, p = 0$	Negative definite	$-y_1^2 - y_2^2 \cdots - y_n^2$	Only negative terms (n terms)
3.	$r = p, p < n$	Positive semi definite	$y_1^2 + y_2^2 + \cdots + y_r^2$	Only positive terms (r terms)
4.	$r < n, p = 0$	Negative semi-definte	$-y_1^2 - y_2^2 - \cdots - y_r^2$	Only negative rerms (r terms)

In all other cases the Q.F is said to be indefinite. Indefinite Q.F contains both positive and negative terms.